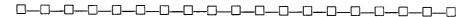


August 25, 2006, 9:00 hrs



During the exam you may use the book, lab manual, copies of sheets and your own notes.

Put your name on all pages which you hand in, and number them. Write the total number of pages you hand in on the first page. Write clearly and not with pencil or red pen. Always motivate your answers. Good luck!

Problem 1. (2.5 pt) Let X be a binary image X and A a structuring element as in Fig. 1.

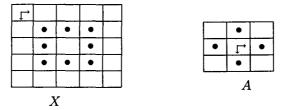


Figure 1: Binary image X and structuring element A.

- **a.** (1 pt) In a similar way to Fig. 1 draw: the dilation $\delta_A(X) = X \oplus A$, the erosion $\varepsilon_A(X) = X \ominus A$, the opening $\gamma_A(X) = X \circ A$ and the closing $\phi_A(X) = X \bullet A$.
- **b.** (0.5 pt) Furthermore, draw $\delta_A \varepsilon_A \delta_A(X)$ and $\varepsilon_A \delta_A \varepsilon_A(X)$.
- c. (1 pt) Prove that for any X, A: $\delta_A \varepsilon_A \delta_A(X) = \delta_A(X)$. Hint: Prove that $\delta_A \varepsilon_A \delta_A(X) \subseteq \delta_A(X)$ and that $\delta_A \varepsilon_A \delta_A(X) \supseteq \delta_A(X)$.

Problem 2. (2 pt) Consider a grey-value image f.

- a. Sobel gradients in the image in horizontal (easterly) direction can be detected by linear filtering using the filter kernel (or mask) in Fig. 2 (left). Give Sobel kernels to detect gradients in northerly, northwesterly, and north-easterly direction.
- **b.** A discrete second derivative filter in the x-direction $\frac{\partial^2}{\partial x^2}$ is defined by convolution with the kernel in Fig. 2(right). If image f is constant, the result of this filter will be zero in every pixel. Show by calculation that the result for an image $f(x,y) = ax^2 + bx + c$, is -2a for each pixel with a,b,c constants.

-1	0	1
-2	0	2
-1	0	1

0	0	0
-1	2	-1
0	0	0

Figure 2: Convolution masks for the Sobel x-gradient filter (left) and the second-order x-derivative filter (right).

Problem 3. (2 pt) Consider the use of snakes to segment a simple grey-scale image given in Fig. 3(a). The aim is to find the contour of the dividing bacterium as shown in Fig. 3(b) (i.e., it need not be split into two parts).

- a. (0.75 pt) Which is the best initialization for an expanding snake (i.e. with a force at right angles to the snake in the outward direction). Discuss why it works in the best case, and how and why it should fail in the others.
- b. (0.5 pt) As (a), but for a contracting snake.
- c. (0.75 pt) At which points of the contour must the smoothness constraint for a snake be relaxed to get the best fit?

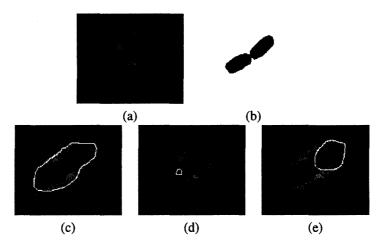


Figure 3: Phase-contrast image of bacterium: (a) original image; (b) ideal object shape; (c), (d) and (e) initial position of snake superimposed on (a) in white.

Problem 4. (2.5 pt) Consider the following inference problem. Given a perspective projection of a cube with three sets of four parallel ribs each, with unknown orientations $\vec{w}^{(X)}$, $\vec{w}^{(Y)}$ and $\vec{w}^{(Z)}$, and three corresponding vanishing points X, Y, Z in the projection plane, see Fig. 4. Two of these points are known $(u_{\infty}^{(Y)}, v_{\infty}^{(Y)}) = (1, 2), (u_{\infty}^{(Z)}, v_{\infty}^{(Z)}) = (0, -2)$. The camera constant f is unknown.

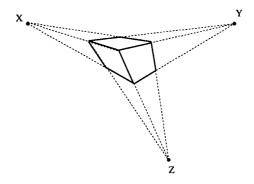


Figure 4: Perspective projection of a cube with three vanishing points.

Compute the three orientation vectors $\vec{w}^{(X)}$, $\vec{w}^{(Y)}$, $\vec{w}^{(Z)}$.

Hint: First compute the camera constant f.